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Is $f_0(1500)$ a Scalar Glueball?

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Abstract

Following the discovery of two new scalar mesons, $f_0(1370)$ and $f_0(1500)$ at the Low Energy Antiproton Ring at CERN, we argue that the observed properties of this pair are incompatible with them both being $Q\overline{Q}$ mesons. We show instead that $f_0(1500)$ is compatible with the ground state glueball expected around 1500 MeV mixed with the nearby states of the 0^{++} $Q\bar{Q}$ nonet. Tests of this hypothesis include the prediction of a further scalar state, $f_0'(1500 - 1800)$ which couples strongly to $K\bar{K}$, $\eta\eta$ and $\eta\eta'$. Signatures for a possible tensor glueball at ~ 2 GeV are also considered.

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1 Introduction

Glueballs are a missing link of the standard model. Whereas the gluon degrees of freedom expressed in L_{QCD} have been established beyond doubt in high momentum data, their dynamics in the strongly interacting limit epitomised by hadron spectroscopy are quite obscure. This may be about to change as a family of candidates for gluonic hadrons (glueballs and hybrids) is now emerging [1, 2]. In this paper we shall argue that scalar mesons around 1.5 GeV, in particular the detailed phenomenology of $f_0(1500)$ and its partner $f_0(1370)$, suggest that a glueball exists in this region, probably mixed with nearby isoscalar members of the scalar nonet. This hypothesis may be tested in forthcoming experiments.

In advance of the most recent data, theoretical arguments suggested that there may be gluonic activity manifested in the 1.5 GeV mass region. Lattice QCD is the best simulation of theory and predicts the lightest "primitive" (ie quenched approximation) glueball to be 0^{++} with mass 1.55 ± 0.05 GeV [3]. Recent lattice computations place the glueball slightly higher in mass at 1.74 ± 0.07 GeV [4] with an optimised value for phenomenology proposed by Teper[5] of 1.57 ± 0.09 GeV. That lattice QCD computations of the scalar glueball mass are now concerned with such fine details represents considerable advance in this field. Whatever the final concensus may be, these results suggest that scalar mesons in the 1.5 GeV region merit special attention. Complementing this has been the growing realisation that there are now too many 0^{++} mesons confirmed for them all to be $Q\overline{Q}$ states [1, 6, 7].

At ~ 1.5 GeV there is a clear 0^{++} signal, $f_0(1500)$, in several experiments [8]-[15], whose serious consideration for being associated with the primitive glueball is enhanced by the fact that its production is by mechanisms traditionally believed to be those that favour gluonic excitations. Specifically these include [16]

1. Radiative J/ψ decay: $J/\psi \rightarrow \gamma + G$ [17]

2. Collisions in the central region away from quark beams and target: $pp \rightarrow p_f(G)p_s$ [12, 15].

3. Proton-antiproton annihilation where the destruction of quarks creates opportunity for gluons to be manifested. This is the Crystal Barrel [8]-[11] and E760 [13, 14] production mechanism in which detailed decay systematics of $f_0(1500)$ have been studied.

4. Tantalising further hints come from the claimed sighting [18] of the $f_0(1500)$ in decays of the hybrid meson candidate [2] $\pi(1800) \rightarrow \pi f_0(1500) \rightarrow \pi\eta\eta$.

The signals appear to be prominent in decay channels such as $\eta\eta$ and $\eta\eta'$ that are traditionally regarded as glueball signatures. However, such experiments are not totally novel and some time ago one of us (FEC) addressed the question of why glueballs had remained hidden during 25 years of establishing the Particle Data Group list [7] of $Q\overline{Q}$ states. This was suggested [16] to be due to the experimental concentration on a restricted

class of production mechanisms and on final states with charged pions and kaons. The more recent emphasis on neutral final states (involving π^0, η, η') was inspired by the possibiblity that η and η' are strongly coupled to glue. This dedicated study of neutrals was a new direction pioneered by the GAMS Collaboration at CERN announcing new states decaying to $\eta\eta$ and $\eta\eta'$ [19].

The Crystal Barrel collaboration at LEAR has made intensive study of $p\bar{p}$ annihilation into neutral final states involving π^0, η and η'. They find a clear signal for $f_0(1500)$ in $\pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$. Our present work extends and generalises the work of ref. [16] in the light of these new data from LEAR. Our purpose is to examine the data on the $f_0(1500)$, compare with predictions for glueballs and identify the seminal experiments now needed to confirm that gluonic degrees of freedom are being manifested in this region. A summary of this work has already been published elsewhere [20].

The structure of the paper is as follows. We first review the experimental data on scalar mesons with special emphasis on states seen in the Crystal Barrel detector at LEAR. We then derive from $\mathrm{SU}(3)_f$ the branching ratios for $Q\overline{Q}$ decays into two pseudoscalars, show that this successfully describes the known decay rates in the 2^{++} $Q\overline{Q}$ nonet, and then compare our predictions to the observed decay modes of $f_0(1500)$. Previous bubble chamber experiments have not observed a $K\overline{K}$ signal in the 1500 MeV mass region which, if confirmed, would imply a set of branching ratios that are unnatural for a state belonging to a quarkonium nonet. If a significant signal were to be observed, it would be possible to find a quarkonium mixing angle that reproduces the observed final state abundances; however the systematics would then imply that $f_0(1500)$ is dominantly $n\bar{n} \equiv (u\bar{u}+d\bar{d})/\sqrt{2}$. This would have two immediate consequences:

1. This would leave the $f_0(1370)$ state, which is also seen in $p\bar{p}$ annihilation with decay branching ratios and total width consistent with an $n\bar{n}$ structure [9, 22], isolated.

2. With either the $f_0(1370)$ or $f_0(1500)$ assigned as the $n\bar{n}$ member, the orthogonal quarkonium in the nonet would have to be dominantly $s\bar{s}$, hence probably heavier than $f_0(1500)$ and decaying strongly into $K\bar{K}$. Identification of this state is now imperative in order to complete the multiplet and discriminate among hypotheses.

We then show that that the decay rates of $f_0(1500)$ are compatible with a glueball state whose mass lies between the $n\bar{n}$ and $s\bar{s}$ scalar quarkonium states, and whose nearby presence disturbs the glueball decays in a characteristic flavour dependent manner. In the climax of the paper we show that dynamics inspired by lattice QCD may be consistent with the data and we consider the implications for glueballs mixing with quarkonia in the 1500 MeV range. We finally show that a reasonable nonet can be constructed with the remaining scalar mesons.

2   Scalar Mesons in the Crystal Barrel

The lowest lying 0^{++} mesons, namely the isospin $I = 0$ $f_0(980)$ and the $I = 1$ $a_0(980)$, have been assumed to be $K\overline{K}$ molecules [23, 24]. This belief is motivated by their strong

couplings to $K\overline{K}$ - in spite of their masses being at the $K\overline{K}$ threshold - and their small $\gamma\gamma$ partial widths. For $f_0(980)$, $\Gamma_{\gamma\gamma} = 0.56 \pm 0.11$ keV [7]. For $a_0(980)$, one finds with a LEAR measurement of the relative branching fraction for a_0 decay to $K\overline{K}$ and $\eta\pi$ [25] the partial width $\Gamma_{\gamma\gamma} = 0.33 \pm 0.13$ keV [1]. Thus the $\gamma\gamma$ partial widths appear to be nearly equal, close to predictions for $K\overline{K}$ molecules (0.6 keV) and much smaller than for $Q\overline{Q}$ states [26].

The nature of these states is likely to be illuminated soon at DAϕNE [24]. If they are not simply $Q\overline{Q}$ then the 0^{++} $Q\overline{Q}$ mesons need to be identified. A new $J^{PC}(I^G) = 0^{++}(1^-)$ meson, $a_0(1450) \to \eta\pi$, has been reported by the Crystal Barrel collaboration at LEAR [25]. This state, with a mass of 1450 ± 40 MeV and a width of 270 ± 40 MeV, appears, together with $a_0(980)$, in the $\eta\pi$ S-wave in $\overline{p}p$ annihilation at rest into $\eta\pi^0\pi^0$. We shall show in section 8 that $a_0(1450)$ can be identified with the $I=1$ member of the ground state 0^{++} $Q\overline{Q}$ nonet. In turn this and the $K^*(1430)$ set the natural energy scale for the scalar nonet.

Recent data in $\overline{p}p$ annihilation at LEAR into $\eta\pi^0\pi^0$ [25], $3\pi^0$ [8, 9] and $\eta\eta\pi^0$ [10, 22] require an $I = 0$ scalar resonance in the range 1320 to 1400 MeV, decaying to $\pi^0\pi^0$ and $\eta\eta$. We shall use 1360 MeV as average mass but shall adopt the nomenclature of the Particle Data Group [7] calling this state $f_0(1370)$. Its width varies between 200 and 700 MeV, depending on theoretical assumptions. For example, the $3\pi^0$ data give $\Gamma \sim 700$ MeV [9], decreasing to 300 ± 80 MeV if the 700 MeV broad background structure [27, 28] centered at 1000 MeV in the $\pi\pi$ S-wave (called $f_0(1300)$ in the latest issue of the Particle Data Group) is introduced in the analysis. This is in good agreement with a coupled channel analysis of $\eta\pi^0\pi^0$, $3\pi^0$ and $\eta\eta\pi^0$ which leads to a mass of 1390 ± 30 MeV and a width of 380 ± 80 MeV [29]. The $\gamma\gamma$ width in this region is also consistent with it containing the $^3P_0(Q\overline{Q})$ state [30, 31].

There is rather general agreement that the ground state $n\bar{n}$ state is manifested here. The debate is one of detail on the relationship of the $f_0(1370)$ to the broad $f_0(1300)$, in particular as to whether these are two independent states or manifestations of a single state, and to what extent unitarity corrections are important [32]. This issue is peripheral to our main analysis which will rely only on the generally accepted association of the ground state $n\bar{n}$ as the seed for the phenomenology in the $f_0(1300 - 1370)$ region and that this is distinct from the $f_0(1500)$ state observed by the Crystal Barrel collaboration decaying to $\pi^0\pi^0$ [8, 9], $\eta\eta$ [10, 22] and $\eta\eta'$ [11]. This state was seen in $\overline{p}p$ annihilation into $3\pi^0$, $\eta\eta\pi^0$ and $\eta\eta'\pi^0$, leading to six final state photons. The masses and widths observed in the three decay channels are consistent, giving the average:

$$(m, \Gamma) = (1509 \pm 10, 116 \pm 17) \text{ MeV}, \tag{1}$$

while the coupled channel analysis [29] gives 1500 ± 10 MeV and the less precise but compatible width of 154 ± 30 MeV. It is possible that $f_0(1500)$ has also been seen by MARKIII and DM2 in $J/\psi \to \gamma + \pi\pi\pi$, hitherto misidentified as 0^{-+} [17] and in $J/\psi \to \gamma\pi\pi$[33]. A resonance decaying to $\pi^0\pi^0$ and $\eta\eta$ was also reported by E760 in $\overline{p}p$ annihilation at higher energies at the Fermilab accumulator, with masses and widths

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(1508, 103) MeV [13] and (1488, 148) MeV [14], respectively. A spin-parity analysis is in progress [34].

The GAMS Collaboration at CERN [19] reports a 0^{++} 180 MeV broad resonance, $f_0(1590)$, decaying to $\eta\eta'$, $\eta\eta$ and $4\pi^0$, also observed in central production [15] and by VES at Serpukhov [35] in $\eta\eta'$: this might be the $f_0(1500)$ state though the status of the $\pi\pi$ branching ratio needs to be clarified.[1] A strong coupling of $f_0(1500)$ to pions would contradict it being primarily an $s\bar{s}$ state and, as we shall argue later, it is a candidate for a glueball mixed with the $Q\bar{Q}$ nonet, where $f_0(1370)$ is dominantly $n\bar{n}$, and a more massive $s\bar{s}$ remains to be identified. The $f_0(1500)$, clearly established in different decay channels and with detailed information on branching ratios to several channels, will form the fulcrum of our investigation.

There are candidates for this $s\bar{s}$ state though their existence and/or $s\bar{s}$ assignment remain to be established. A 0^{++} structure, $f_0'(1525)$, with poorly known width (~ 90 MeV) is observed to decay into $K_S K_S$ in $K^- p$ interactions [37]. This state requires confirmation from other experiments. The $\theta(1690)$ (now known as $f_J(1720)$ [7]) is a candidate due to its affinity for $K\bar{K}$ and $\eta\eta$ decays though its quantum numbers, 0^{++} or 2^{++}, are still controversial.

An $I = 0$ scalar with a width of 56 MeV, is observed at 1446 MeV by the WA91 collaboration at CERN in pp central collisions [12]. It decays to four pions, dominantly through $\rho^0(\pi^+\pi^-)_P$ where the dipion is in a P-wave. This may be the same as $f_0(1500)$ produced in the second of the favoured glueball mechanisms (section 1), with its apparent small width being the result of interference between $f_0(1370)$ and $f_0(1500)$ [38]. In any event it does not detract from the qualitative observation that there are too many isoscalars observed in various production mechanisms for them all to be explained naturally within a $Q\bar{Q}$ picture. The fact that there does not appear to be such copious activity in the $I = 1$ and strange sectors adds weight to the suspicion that glueball excitation is affecting the $I = 0$ spectrum.

A substantial part of this paper will examine what the flavour content of two-body decays can reveal about the structure of the initial meson. Based on this analysis we shall argue that the Crystal Barrel $f_0(1500)$ has decay properties incompatible with a $Q\overline{Q}$ state. In addition we shall show that a reasonable $Q\overline{Q}$ nonet may be constructed with $a_0(1450)$, $f_0(1370)$, $K_0^*(1430)$, and an $s\overline{s}$ state above 1500 MeV, thus leaving $f_0(980)$ and $f_0(1500)$ as exotic (not simply $Q\overline{Q}$) states.

3 Quarkonium Decay Amplitudes

Consider a quarkonium state

$$|Q\overline{Q}\rangle = \cos\alpha|n\bar{n}\rangle - \sin\alpha|s\bar{s}\rangle \tag{2}$$

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[1]The $\pi\pi$ branching ratio is the largest deviation but is "not in contradiction" with the $f_0(1590)$ of GAMS and the $f_0(1500)$ of Crystal Barrel being the same state[36]

where

$$n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}. \tag{3}$$

The mixing angle α is related to the usual nonet mixing angle θ [7] by the relation

$$\alpha = 54.7° + \theta. \tag{4}$$

For $\theta = 0$ the quarkonium state becomes pure $SU(3)_f$ octet, while for $\theta = \pm90°$ it becomes pure singlet. Ideal mixing occurs for $\theta = 35.3°$ (-54.7°) for which the quarkonium state becomes pure $s\bar{s}$ ($\bar{n}n$).

In general we define

$$\eta = \cos\phi|n\bar{n}\rangle - \sin\phi|s\bar{s}\rangle \tag{5}$$

and

$$\eta' = \sin\phi|n\bar{n}\rangle + \cos\phi|s\bar{s}\rangle \tag{6}$$

with $\phi = 54.7° + \theta_{PS}$, where θ_{PS} is the usual octet-singlet mixing angle in $SU(3)_f$ basis where

$$\eta = \cos(\theta_{PS})|\eta_8\rangle - \sin(\theta_{PS})|\eta_1\rangle, \tag{7}$$

$$\eta' = \sin(\theta_{PS})|\eta_8\rangle + \cos(\theta_{PS})|\eta_1\rangle. \tag{8}$$

The decay of quarkonium into a pair of mesons $Q\bar{Q} \to M(Q\bar{q}_i)M(q_i\bar{Q})$ involves the creation of $q_i\bar{q}_i$ from the vacuum. If the ratio of the matrix elements for the creation of $s\bar{s}$ versus $u\bar{u}$ or $d\bar{d}$ is denoted by [2]

$$\rho \equiv \frac{\langle 0|V|s\bar{s}\rangle}{\langle 0|V|d\bar{d}\rangle}, \tag{9}$$

then the decay amplitudes of an isoscalar 0^{++} (or 2^{++}) are proportional to

$$
\begin{aligned}
\langle Q\bar{Q}|V|\pi\pi\rangle &= \cos\alpha \\
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \cos\alpha(\rho - \sqrt{2}\tan\alpha)/2 \\
\langle Q\bar{Q}|V|\eta\eta\rangle &= \cos\alpha(1 - \rho\sqrt{2}\tan\alpha)/2 \\
\langle Q\bar{Q}|V|\eta\eta'\rangle &= \cos\alpha(1 + \rho\sqrt{2}\tan\alpha)/2.
\end{aligned}
\tag{10}
$$

The corresponding decay amplitudes of the isovector are

$$
\begin{aligned}
\langle Q\bar{Q}|V|K\bar{K}\rangle &= \rho/2 \\
\langle Q\bar{Q}|V|\pi\eta\rangle &= 1/\sqrt{2} \\
\langle Q\bar{Q}|V|\pi\eta'\rangle &= 1/\sqrt{2},
\end{aligned}
\tag{11}
$$

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[2]we shall assume that $\langle 0|V|d\bar{d}\rangle \equiv \langle 0|V|u\bar{u}\rangle$.

and those for K^* decay

$$\begin{aligned}
\langle Q\overline{Q}|V|K\pi\rangle &= \sqrt{3}/2 \\
\langle Q\overline{Q}|V|K\eta\rangle &= (\sqrt{2}\rho - 1)/\sqrt{8} \\
\langle Q\overline{Q}|V|K\eta'\rangle &= (\sqrt{2}\rho + 1)/\sqrt{8}.
\end{aligned}$$
$$(12)$$

For clarity of presentation we have presented eqn. 10,11 and 12 in the approximation where $\eta \equiv (n\bar{n} - s\bar{s})/\sqrt{2}$ and $\eta' \equiv (n\bar{n} + s\bar{s})/\sqrt{2}$, i.e. for a pseudoscalar mixing angle $\theta_{PS} \sim -10°$ ($\phi = 45°$). This is a useful mnemonic; the full expressions for arbitrary η, η' mixing angles θ_{PS} are given in appendix A and are used in detailed comparisons throughout this paper. Exact $SU(3)_f$ flavour symmetry corresponds to $\rho = 1$; empirically $\rho \geq 0.8$ for well established nonets such as 1^{--} and 2^{++} [21, 39].

The partial width into a particular meson pair $M_i M_j$ may be written as

$$\Gamma_{ij} = c_{ij}|M_{ij}|^2 \times |F_{ij}(\vec{q})|^2 \times p.s.(\vec{q}) \equiv \gamma_{ij}^2 \times |F_{ij}(\vec{q})|^2 \times p.s.(\vec{q}) \qquad (13)$$

where $p.s.(\vec{q})$ denotes the phase-space, $F_{ij}(\vec{q})$ are model-dependent form factors, M_{ij} is the relevant amplitude (eqn. 10,11 or 12) and c_{ij} is a weighting factor arising from the sum over the various charge combinations, namely 4 for $K\bar{K}$, 3 for $\pi\pi$, 2 for $\eta\eta'$ and 1 for $\eta\eta$ for isoscalar decay (eqn. 10), 4 for $K\bar{K}$, 2 for $\pi\eta$ and 2 for $\pi\eta'$ for isovector decay (eqn. 11) and 2 for K^* decays (eqn. 12). The dependence of $\gamma_{ij}^2 = c_{ij}|M_{ij}|^2$ upon the mixing angle α is shown in fig. 1a for the isoscalar decay in the case of $SU(3)_f$ symmetry, $\rho = 1$.

We confront the above with data on the established 2^{++} nonet, determine a probable range of values for ρ, and then compare with $f_0(1500)$ decays.

4 Flavour Symmetry in Meson Decays

4.1 $Q\overline{Q}$ Decays: the 2^{++} nonet

Exact $SU(3)_f$ flavour symmetry requires the parameter ρ to be unity. To get a feeling for symmetry breaking in the $Q\overline{Q}$ sector we have computed some of the expected branching ratios for the tensor mesons $a_2(1320)$, $f_2(1270)$ and $K_2^*(1430)$ decaying to two pseudoscalars and have compared them with data [7]. The decay branching ratio B of a $Q\overline{Q}$ state is proportional to the partial width (eqn. 13). We use for the phase space factor $p.s.(\vec{q}) = q$ and for the formfactor

$$|F_{ij}(\vec{q})|^2 = q^{2\ell} \times exp(-q^2/8\beta^2), \qquad (14)$$

where $\ell = 2$ is the angular momentum in the final state with daughter momenta q and $\beta \simeq 0.4$ GeV/c [39]. The ratios of the various partial widths are rather insensitive to choice among different successful descriptions of meson spectroscopy and dynamics. The detailed sensitivity to form factors is discussed in appendix B.

The pseudoscalar mixing angle θ_{PS} has the empirical value $-(17.3 \pm 1.8)°$ [40] (and hence $\phi = 37.4°$, somewhat removed from the "ideal" $45°$ used in section 3). We shall use the full expressions given in appendix A and the above value of θ_{PS} in all phenomenology. Analogously, for the tensor mixing angle, $\theta = 26°$ [7]. The predictions are fitted to the experimental values [7]

$$
\begin{aligned}
B(f_2 \to \eta\eta)/B(f_2 \to \pi\pi) &= (5.3 \pm 1.2) \times 10^{-3}, \\
B(f_2 \to K\overline{K})/B(f_2 \to \pi\pi) &= (5.4 \pm 0.6) \times 10^{-2}, \\
B(a_2 \to \eta\pi)/B(a_2 \to K\overline{K}) &= 2.95 \pm 0.54, \\
B(a_2 \to \eta'\pi)/B(a_2 \to K\overline{K}) &= 0.116 \pm 0.029.
\end{aligned}
\tag{15}
$$

Figure 2 shows the χ^2 distribution as a function of ρ for various values of β. The distribution does not change significantly for $\beta >1$ GeV/c. A good fit (with a χ^2 confidence level of more than 5%) is obtained with $\beta = 0.5$ GeV/c (or larger) for which $\rho = 0.96 \pm 0.04$. This result is consistent with $K_2^*(1430)$ decays although the experimental errors are large: $B(K_2^* \to K\pi)/B(K_2^* \to K\eta) = (3.6^{+7.1}_{-2.3}) \times 10^3$ [7]. We therefore conclude that flavour symmetry breaking effects cannot be large in this established $Q\overline{Q}$ nonet. Similar conclusions follow for a wide range of $Q\overline{Q}$ decays (at least in the $\beta \to \infty$ limit where form factors are ignored [21]). Thus it seems reasonable to expect that for $Q\overline{Q}$ scalar decays also, $\rho \approx 1$ and $\beta \sim 0.5$ GeV/c.

We have refrained from using the corresponding ratios for $f_2'(1525)$ decays since the $\pi\pi$ decay width is (i) poorly known and (ii) very sensitive to the precise tensor nonet mixing angle θ. For the ratio $B(f_2' \to \eta\eta)/B(f_2' \to K\overline{K})$ we find 0.07 for $\rho = 1$ and $\beta = 0.5$ GeV/c, in good agreement with experiments (0.11 ± 0.04) [7] but at variance with the value (0.39 ± 0.05) advocated by the Particle Data Group which relies on one experiment only.

4.2   The Decay Properties of $f_0(1500)$

The branching ratios for $f_0(1500)$ production and decay are [9, 11, 22]:

$$
\begin{aligned}
B(\overline{p}p \to f_0\pi^0, f_0 \to \pi^0\pi^0) &= (8.1 \pm 2.8) \times 10^{-4}, \\
B(\overline{p}p \to f_0\pi^0, f_0 \to \eta\eta) &= (5.5 \pm 1.3) \times 10^{-4}, \\
B(\overline{p}p \to f_0\pi^0, f_0 \to \eta\eta') &= (1.6 \pm 0.4) \times 10^{-4}.
\end{aligned}
\tag{16}
$$

where the errors do not reflect the statistical significance of the signals, but rather uncertainties in the various assumptions made in the fitting procedures. The decay branching ratios are given by eqn. 13 with $p.s.(\vec{q}) = q$ and the form factors

$$
|F_{ij}(\vec{q})|^2 = \exp(-q^2/8\beta^2),
\tag{17}
$$

since $\ell = 0$ for 0^{++} decays to two pseudoscalars. The $f_0(1500)$ observed in $\eta\eta'$ decay has a mass of 1545 ± 25 MeV and lies just above threshold [11]. We use for q the average decay momentum (194 MeV/c) derived from the damped Breit-Wigner function used in the analysis of ref. [11]. The uncertainty in the mass ($\pm$ 25 MeV) is taken into account when computing the error on γ^2. With $\beta = 0.5$ GeV/c (section 4.1) we find:

$$R_1 = \frac{\gamma^2(f_0(1500) \to \eta\eta)}{\gamma^2(f_0(1500) \to \pi\pi)} = 0.27 \pm 0.11, \tag{18}$$

$$R_2 = \frac{\gamma^2(f_0(1500) \to \eta\eta')}{\gamma^2(f_0(1500) \to \pi\pi)} = 0.19 \pm 0.08, \tag{19}$$

where $\pi\pi$ includes $\pi^+\pi^-$. These results are in good agreement with the results of the coupled channel analysis [29]. A signal for scalar decay to $K\overline{K}$ has not yet been observed in $\overline{p}p$ annihilation in the the 1500 MeV region. A bubble chamber experiment [41] reports $B.R.(p\bar{p} \to X\pi; X \to K\bar{K}) < 3.4 \times 10^{-4}$ which interpreted directly as an intensity leads to a (90% C.L.) upper limit:

$$R_3 = \frac{\gamma^2(f_0(1500) \to K\overline{K})}{\gamma^2(f_0(1500) \to \pi\pi)} \lesssim 0.1. \tag{20}$$

However, interference effects among amplitudes could lead to an underestimate for this number. We shall consider the implications of the above R_3 value but shall also give results allowing for larger values.

We find that if in the decay of some state the ratios of partial widths (per charge combination and after phase space and form factor corrections) for $\eta\eta/\pi\pi$ and $\eta\eta/K\overline{K}$ are simultaneously both greater than unity, then this state cannot be a quarkonium decay unless $s\bar{s}$ production is enhanced ($\rho > 1$) (see fig. 3). The $f_0(1500)$ data on $\eta\eta/K\bar{K}$ satisfy this, but the $\eta\eta/\pi\pi$ is inconclusive; at 1σ the ratio per charge configuration gives 0.81 ± 0.33. If $f_0(1590)$ and $f_0(1500)$ are the same state, as discussed above (see footnote 1), then if the $\pi\pi$ branching ratio is reduced towards the GAMS limit[15, 19] the value of R_1 would rise such that it may be possible to confirm the $f_0(1500)$ as a glueball by this test alone. We cannot overemphasise the importance of a mutually consistent analysis of the data on these experiments, in particular for clarifying the magnitude of the $\eta\eta/\pi\pi$ ratio. If the ratio rises, as for GAMS, it would immediately point towards a glueball; if the ratio remains as in eqn.18 then the arguments are less direct but there still appears not to be a consistent $Q\bar{Q}$ solution to the flavour dependence of the ratios of partial widths and the magnitudes of the total widths for the $f_0(1500) - f_0(1370)$ system.

Figure 1 shows the invariant couplings γ^2 as a function of α for $\rho = 1$ (a) and $\rho = 0.75$ and 1.25 (b), for a pseudoscalar mixing angle $\theta = -17.3°$ [40]. The effects of SU(3)$_f$ breaking, $\rho < 1$, in the region where $n\bar{n}$ dominates the Fock state ($0 \leq \alpha \leq 30°$) are interesting (fig. 1b). We see that the branching ratios for η or η' are little affected (essentially because they are produced via the $\overline{n}n$ component in the η which is ρ independent) whereas $K\bar{K}$ depends on ρ (due to $s\bar{s}$ creation triggering $K\bar{K}$ production from an $n\bar{n}$ initial state). Thus we can suppress $K\overline{K}$ by letting $\rho \to 0$ without affecting the $\eta\eta/\pi\pi$

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ratio substantially. In this case the measured values for $\eta\eta$ and $\eta\eta'$ and the upper limit for $K\overline{K}$ suggest that $\alpha \sim 0$, hence $f_0(1500)$ is a pure $\overline{n}n$ meson. However this is still unsatisfactory as the required value of ρ implies a dramatic suppression of $s\bar{s}$ creation to a degree not seen elsewhere in hadron decays.

Figure 4 shows the dependence of $\rho \tan\alpha$ (eqn. 10) on R_1 and R_2. From the experimental values (eqn. 18 and 19) we find consistency for $\rho \tan(\alpha) \sim$ -0.1. Hence either ρ is small or α is small. The former leads to unacceptable violation of $SU(3)_f$ and the latter predicts the ratio $\gamma^2(K\overline{K})/\gamma^2(\pi\pi)$ to be $1/3$.

Form factors of the type in eqn. 17 tend to kill transitions at large q, for example $F_{\eta\eta}(q_\eta)/F_{\pi\pi}(q_\pi) > 1$ (i.e. opposite to naive phase space which grows with q). Without a form factor, e.g. $|F_{ij}(\vec{q})| \equiv 1$, we obtain $R_1 = 0.31 \pm .13$, $R_2 = 0.25 \pm 0.11$, which excludes a common range of $\rho \tan(\alpha)$ (see fig. 4), and $R_3 < 0.12$. The form factors used in ref. [44] have a node at $q \sim 0.9$ GeV/c and hence lead to an even stronger suppression of the observed $\pi\pi$ intensity which dramatically reduces R_3, in contradiction with the expected $1/3$ for an $\overline{n}n$ state (see also appendix B).

In fig. 5 we plot the allowed regions of ρ vs. α. The grey area shows the common values of ρ and α which satisfy the Crystal Barrel data each at the 90 % C.L. while the black area shows the restricted range allowed by $K\overline{K}$ (eqn. 20).

If one wishes to force $f_0(1500)$ into a $Q\overline{Q}$ nonet, then independent of form factors and $SU(3)_f$ breaking one is forced to $\alpha \to 0$, whereby $f_0(1500)$ has strong $n\bar{n}$ content. This remains true even were the $K\bar{K}$ branching ratio, currently being remeasured at LEAR, significantly greater than the R_3 value of eqn.20: the magnitude of the $K\bar{K}/\pi\pi$ ratio is controlled more by flavour symmetry breaking than by the magnitude of the $n\bar{n}$ - $s\bar{s}$ mixing angle in the $\alpha \to 0$ region. This immediately implies that the orthogonal isoscalar member will be dominantly $s\bar{s}$, with mass above 1500 MeV and prominent in $K\bar{K}$. The $f_0'(1525)$ if confirmed or the "θ" ($f_J(1720)$), if 0^{++} , could be this state. However, the $f_0(1370)$, seen in both $\pi\pi, \eta\eta$ is then left in isolation.

In the next sections we confront the data on $G = f_0(1500)$ with the extreme hypothesis that it is dominantly a glueball. Its production in the canonical glue enhanced environments of $J/\psi \to \gamma(G \to 4\pi)$ [17], $pp \to p(G)p$ [12], and $p\bar{p}$ annihilation [8, 9, 10, 11, 13, 14] is consistent with this hypothesis, and recent lattice QCD studies [3, 4] predict that a scalar glueball exists in this region of mass. It thus scores well on two of the three glueball figures of merit [16].

We shall now consider the dynamics and phenomenology of glueball decays.

5 Primitive Glueball Decays

The decays of $c\bar{c}$, in particular $\chi_{0,2}$, provide a direct window on G dynamics in the $0^{++}, 2^{++}$ channels insofar as the hadronic decays are triggered by $c\bar{c} \to gg \to Q\bar{Q}Q\bar{Q}$ (fig. 6a). It is necessary to keep in mind that these are in a different kinematic region to that appropriate to our main analysis but, nonetheless, they offer some insights into the gluon dynamics. Mixing between hard gluons and 0^{++}, 2^{++} $Q\bar{Q}$ states (fig. 6c) is

improbable at these energies as the latter 1 - 1.5 GeV states will be far off their mass-shell. Furthermore,the narrow widths of $\chi_{0,2}$ are consistent with the hypothesis that the 3.5 GeV region is remote from the prominent $0^+, 2^+$ glueballs, G. Thus we expect that the dominant decay dynamics is triggered by hard gluons directly fragmenting into two independent $Q\bar{Q}$ pairs (fig. 6a) or showering into lower energy gluons (fig. 6b). We consider the former case now; mixing with $Q\bar{Q}$ (fig. 6c) and $G \to GG$ (fig. 6b) will be discussed in section 6.

$G \to QQ\bar{Q}\bar{Q}$

This was discussed in ref. [16] and the relative amplitudes for the process shown in fig. 6a read

$$
\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \\
\langle G|V|K\overline{K}\rangle &= R \\
\langle G|V|\eta\eta\rangle &= (1+R^2)/2 \\
\langle G|V|\eta\eta'\rangle &= (1-R^2)/2,
\end{aligned}
\tag{21}
$$

with generalizations for arbitrary pseudoscalar mixing angles given in appendix A and where $R \equiv \langle g|V|s\bar{s}\rangle/\langle g|V|d\bar{d}\rangle$. SU(3)$_f$ symmetry corresponds to $R^2 = 1$. In this case the relative branching ratios (after weighting by the number of charge combinations) for the decays $\chi_{0,2} \to \pi\pi, \eta\eta, \eta\eta', K\overline{K}$ would be in the relative ratios $3 : 1 : 0 : 4$. Data for χ_0 are in accord with this where the branching ratios are (in parts per mil) [7]:

$$
\begin{aligned}
B(\pi^0\pi^0) &= 3.1 \pm 0.6 \\
\frac{1}{2}B(\pi^+\pi^-) &= 3.7 \pm 1.1 \\
\frac{1}{2}B(K^+K^-) &= 3.5 \pm 1.2 \\
B(\eta\eta) &= 2.5 \pm 1.1.
\end{aligned}
\tag{22}
$$

No signal has been reported for $\eta\eta'$. Flavour symmetry is manifested in the decays of χ_2 also:

$$
\begin{aligned}
B(\pi^0\pi^0) &= 1.1 \pm 0.3 \\
\frac{1}{2}B(\pi^+\pi^-) &= 0.95 \pm 0.50 \\
\frac{1}{2}B(K^+K^-) &= 0.75 \pm 0.55 \\
B(\eta\eta) &= 0.8 \pm 0.5,
\end{aligned}
\tag{23}
$$

again in parts per mil. The channel $\eta\eta'$ has not been observed either. These results are natural as they involve hard gluons away from the kinematic region where G bound states dominate the dynamics. If glueballs occur at lower energies and mix with nearby $Q\bar{Q}$ states, this will in general lead to a distortion of the branching ratios from the "ideal" equal weighting values above (a detailed discussion of this follows in section 6.1), and also in causing significant mixing between $n\bar{n}$ and $s\bar{s}$ in the quarkonium eigenstates. Conversely, "ideal" nonets, where the quarkonium eigenstates are $n\bar{n}$ and $s\bar{s}$, are expected to signal those J^{PC} channels where the masses of the prominent glueballs are remote from those of the quarkonia.

An example of this is the 2^{++} sector where the quarkonium members are "ideal". Data on glue in the 2^{++} channel, and potential mixing of glue with $n\bar{n}/s\bar{s}$, may be probed by $J/\psi \rightarrow \gamma + f_2(1270)/f_2'(1525)$ which measures the $gg \rightarrow n\bar{n}/s\bar{s}$ amplitude (fig. 6a) insofar as $J/\psi \rightarrow \gamma + gg$ mediates these channels. The branching ratios in parts per mil are

$$\frac{1}{2}B(J/\psi \rightarrow \gamma f_2(1270)) = 0.69 \pm 0.07$$
$$B(J/\psi \rightarrow \gamma f_2(1525)) = 0.63 \pm 0.1.$$
$$(24)$$

Here again there is no sign of significant symmetry breaking. Furthermore we note the ideal $n\bar{n}$ and $s\bar{s}$ nature of the 2^{++}, manifested both by the masses and the flavour dependence of the branching ratios, which suggests that G mixing is nugatory in this channel. These data collectively suggest that prominent 2^{++} glueballs are not in the $1.2 - 1.6$ GeV region which in turn is consistent with lattice calculations where the mass of the 2^{++} primitive glueball is predicted to be larger than 2 GeV. The sighting of a 2^{++} state in the glueball favoured central production, decaying into $\eta\eta$ with no significant $\pi\pi$[42] could be the first evidence for this state. In view of our earlier remarks on the $\eta\eta/\pi\pi$ and $\eta\eta/K\bar{K}$ ratios being a potentially direct signature for a glueball, we recommend that a detailed search now be made for this state in $\eta\eta$ **and** $K\bar{K}$ (and $\pi\pi$) channels in central production.

The phenomenology of the $J^{PC} = 0^{++}$ sector in the 1.2 - 1.6 GeV region is rather different to this: the $f_0(1500)$ - $f_0(1370)$ system cannot be described within a $Q\bar{Q}$ nonet, nor do the decay branching ratios of the $f_0(1500)$ respect the flavour blindness of glue, (eqn. 19 and 20). We shall now begin to focus on this problem.

It was shown earlier [16] that violation of flavour symmetry ($R^2 \neq 1$) leads to smaller $K\bar{K}/\eta\eta$ and a finite $\eta\eta'$, at least if graph 6a dominates glueball decay. This follows immediately from eqn. 21 or from the generalized formulae given in appendix A. The contributions of graph 6a are shown in fig. 7a as a function of R^2. Graph 6a becomes compatible with the Crystal Barrel data and the small $K\bar{K}$ ratio if $|R| \sim 0.3$ [1] - a rather strong violation of symmetry which might be suggestive of significant mixing between G and flavoured states. If R_3 were as large as $\frac{1}{3}$ (the value for an $n\bar{n}$ state) an attempt to interpret as gluonium would still require $|R|$ to be as small as 0.5. We now show that

mixing of G and $Q\bar{Q}$ is to be expected if the strong coupling picture of QCD, as in the lattice, is a guide to their dynamics.

6 $Q\bar{Q}$ and Glueball Decays in Strong Coupling QCD

In the strong coupling ($g \to \infty$) lattice formulation of QCD, hadrons consist of quarks and flux links, or flux tubes, on the lattice. "Primitive" $Q\bar{Q}$ mesons consist of a quark and antiquark connected by a tube of coloured flux whereas primitive glueballs consist of a loop of flux (fig. 8a,b) [43]. For finite g these eigenstates remain a complete basis set for QCD but are perturbed by two types of interaction [44]:

1. V_1 which creates a Q and a $\bar{Q}$ at neighbouring lattice sites, together with an elementary flux-tube connecting them, as illustrated in fig. 8c,

2. V_2 which creates or destroys a unit of flux around any plaquette (where a plaquette is an elementary square with links on its edges), illustrated in fig. 8d.

The perturbation V_1 in leading order causes decays of $Q\bar{Q}$ (fig. 8e) and also induces mixing between the "primitive" glueball (G_0) and $Q\bar{Q}$ (fig. 8f). It is perturbation V_2 in leading order that causes glueball decays and leads to a final state consisting of $G_0 G_0$ (fig. 8g); decays into $Q\bar{Q}$ pairs occur at higher order, by application of the perturbation V_1 twice. This latter sequence effectively causes G_0 mixing with $Q\bar{Q}$ followed by its decay. Application of V_1^2 leads to a $Q^2\bar{Q}^2$ intermediate state which then turns into colour singlet mesons by quark rearrangement (fig. 6a); application of V_2 would lead to direct coupling to glue in η, η' or $V_2 \times V_1^2$ to their $Q\overline{Q}$ content (fig. 6b).

The absolute magnitudes of these various contributions require commitment to a detailed dynamics and are beyond the scope of this first survey. We concentrate here on their **relative** contributions to the various two body pseudoscalar meson final states available to 0^{++} meson decays.For $Q\bar{Q} \to Q\bar{q}q\bar{Q}$ decays induced by V_1, the relative branching ratios are given in eqn. 10 where one identifies

$$\rho \equiv \frac{\langle Q\bar{s}s\bar{Q}|V_1|Q\bar{Q}\rangle}{\langle Q\bar{d}d\bar{Q}|V_1|Q\bar{Q}\rangle}. \tag{25}$$

The magnitude of ρ and its dependence on J^{PC} is a challenge for the lattice. We turn now to consider the effect of V_1 on the initial "primitive" glueball G_0. Here too we allow for possible flavour dependence and define

$$R^2 \equiv \frac{\langle s\bar{s}|V_1|G_0\rangle}{\langle d\bar{d}|V_1|G_0\rangle}. \tag{26}$$

The lattice may eventually guide us on this magnitude and also on the ratio R^2/ρ. In the absence of this information we shall leave R as free parameter and set $\rho = 1$.

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6.1 Glueball-$Q\overline{Q}$ mixing at $O(V_1)$

In this first orientation we shall consider mixing between G_0 (the primitive glueball state) and the quarkonia, $n\bar{n}$ and $s\bar{s}$, at leading order in V_1 but will ignore that between the two different quarkonia which is assumed to be higher order perturbation.

The mixed glueball state is then

$$G = |G_0\rangle + \frac{|n\bar{n}\rangle\langle n\bar{n}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}} + \frac{|s\bar{s}\rangle\langle s\bar{s}|V_1|G_0\rangle}{E_{G_0} - E_{s\bar{s}}} \tag{27}$$

which may be written as

$$G = |G_0\rangle + \frac{\langle n\bar{n}|V_1|G_0\rangle}{\sqrt{2}(E_{G_0} - E_{n\bar{n}})}\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} \tag{28}$$

where

$$\omega \equiv \frac{E_{G_0} - E_{n\bar{n}}}{E_{G_0} - E_{s\bar{s}}} \tag{29}$$

is the ratio of the energy denominators for the $n\bar{n}$ and $s\bar{s}$ intermediate states in old fashioned perturbation theory (fig. 6d).

Denoting the dimensionless mixing parameter by

$$\xi \equiv \frac{\langle d\bar{d}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}}, \tag{30}$$

the three eigenstates become, to leading order in the perturbation,

$$N_G|G\rangle = |G_0\rangle + \xi\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} \equiv |G_0\rangle + \sqrt{2}\xi|Q\overline{Q}\rangle$$
$$N_s|\Psi_s\rangle \equiv |s\bar{s}\rangle - \xi R^2\omega|G_0\rangle$$
$$N_n|\Psi_n\rangle \equiv |n\bar{n}\rangle - \xi\sqrt{2}|G_0\rangle$$
$$\tag{31}$$

with the normalizations

$$N_G = \sqrt{1 + \xi^2(2 + \omega^2 R^4)},$$
$$N_s = \sqrt{1 + \xi^2\omega^2 R^4},$$
$$N_n = \sqrt{1 + 2\xi^2}. \tag{32}$$

Recalling our definition of quarkonium mixing

$$|Q\overline{Q}\rangle = \cos\alpha|n\bar{n}\rangle - \sin\alpha|s\bar{s}\rangle \tag{33}$$

we see that G_0 has mixed with an effective quarkonium of mixing angle α where $\sqrt{2}\tan\alpha = -\omega R^2$ (eqn . 10). For example, if $\omega R^2 \equiv 1$, the SU(3)$_f$ flavour symmetry maps a glueball onto quarkonium where $\tan\alpha = -1/\sqrt{2}$ hence $\theta = -90°$, leading to the familiar flavour singlet

$$|Q\overline{Q}\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}. \qquad (34)$$

When the glueball is far removed in mass from the $Q\bar{Q}$, $\omega \to 1$ and flavour symmetry ensues; the $\chi_{0,2}$ decay and the 2^{++} analysis of sections 4 and 5 are examples of this "ideal" situation. However, when $\omega \neq 1$, as will tend to be the case when G_0 is in the vicinity of the primitive $Q\bar{Q}$ nonet (the 0^{++} case of interest here), significant distortion from naive flavour singlet can arise.

If the G_0 component contributed negligibly to the decays, the expectations would be that there is (eqn. 31) (i) a state $\Psi_n \to \pi\pi, \eta\eta, K\bar{K}$ which is compatible with $f_0(1370)$; (ii) a state $\Psi_s \to K\bar{K}, \eta\eta, \eta\eta'$, but not $\pi\pi$, to be established; (iii) the 1500 MeV state G for which the decay amplitudes relative to $\pi\pi$ are (replacing $\sqrt{2}\tan\alpha$ by $-\omega R^2$ in eqn. 10),

$$
\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \\
\langle G|V|K\overline{K}\rangle &= (\rho + \omega R^2)/2 \\
\langle G|V|\eta\eta\rangle &= (1 + \omega\rho R^2)/2 \\
\langle G|V|\eta\eta'\rangle &= (1 - \omega\rho R^2)/2,
\end{aligned}
\qquad (35)
$$

with generalisation given in appendix A. The invariant decay couplings γ_{ij}^2 are shown in fig. 7c as a function of ωR^2 for $\rho = 1$. Thus, for example, SU(3)$_f$ may be exact for the glue-quark coupling ($R = 1$) but mass breaking effects ($\Delta m \equiv m_s - m_d \neq 0$) can cause dramatic effects if $E_G - E_{n\bar{n}}$ or $E_G - E_{s\bar{s}}$ is accidentally small, such that $\omega \to 0$ or ∞ respectively. We now consider an explicit mixing scheme motivated by three mutually consistent phenomenological inputs:

1. The suppression of $K\bar{K}$ in the $f_0(1500)$ decays suggests a destructive interference between $n\bar{n}$ and $s\bar{s}$ such that $\omega R^2 < 0$ (see fig. 7c). This arises naturally if the primitive glueball mass is between those of $n\bar{n}$ and the primitive $s\bar{s}$. As the mass of $G_0 \to m_{n\bar{n}}$ or $m_{s\bar{s}}$, the $K\bar{K}$ remains suppressed though non-zero; thus eventual quantification of the $K\bar{K}$ signal will be important.

2. Lattice QCD suggests that the "primitive" scalar glueball G_0 lies at or above 1500 MeV, hence above the $I = 1$ $Q\bar{Q}$ state $a_0(1450)$ and the (presumed) associated $n\bar{n}$ $f_0(1370)$. Hence $E_{G_0} - E_{n\bar{n}} > 0$ in the numerator of ω.

3. The $\Delta m = m_{s\bar{s}} - m_{n\bar{n}} \approx 200 - 300$ MeV suggests that the primitive $s\bar{s}$ state is in the region 1600-1700 MeV. This is consistent with the requirement from (1) and (2) that $m_{n\bar{n}} < m_{G_0} < m_{s\bar{s}}$.

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Higher order perturbation effects will be required for a complete treatment, in particular including mixing between $n\bar{n}$ and $s\bar{s}$, but that goes beyond this first orientation and will require more data to constrain the analysis. We shall present a posteriori evidence supporting this leading order approximation.

Tests of this scenario and its further development will follow as the predicted states are isolated and the flavour dependence of their branching ratios is measured. In order to compute the decay branching ratios of the physical (mixed) states, we need to incorporate the contributions from the primitive glueball components, G_0. We consider this now.

6.2 $G_0 \to G_0 G_0$ at $O(V_2)$

Here the glueball decays directly into pairs of glueballs or mesons whose Fock states have strong overlap with gg (fig. 6b). This topology will not feed final states such as $\pi\pi$ nor $K\bar{K}$ since gluons are isoscalar. To the extent that there is significant G coupling to η, η' or to the $\pi\pi$ S-wave, $(\pi\pi)_s$, (e.g. $\psi' \to \psi\eta$ and $\psi(\pi\pi)_s$ each have large intrinsic couplings notwithstanding the fact that they are superficially OZI violating) one may anticipate $\eta\eta, \eta\eta'$, and $(\pi\pi)_s(\pi\pi)_s$ in the decays of scalar glueballs. Analogously for 0^{-+} glueballs one may anticipate $\eta(\pi\pi)_s$ or $\eta'(\pi\pi)_s$ decays.

The manifestation of this mechanism in final states involving the η or η' mesons depends on the unknown overlaps such as $\langle gg|V|q\bar{q}\rangle$ in the pseudoscalars. We consider various possibilities from the literature without prejudice at this stage.

In the limit $m_{u,d} \to 0$ chiral symmetry suggests that the direct coupling of glue to the η or η' occurs dominantly through their $s\bar{s}$ content, thereby favouring the η'. This argument has been applied to the $\eta(1460)$ in refs.[45, 46, 47]:

$$\frac{\langle gg|V|\eta'\rangle}{\langle gg|V|\eta\rangle} = \frac{\langle s\bar{s}|\eta'\rangle}{\langle s\bar{s}|\eta\rangle} = \frac{cot(\phi) + \sqrt{2}\lambda}{\lambda\sqrt{2}cot(\phi) - 1}, \tag{36}$$

where

$$\lambda \equiv \frac{\langle gg(0^-)|V|d\bar{d}\rangle}{\langle gg(0^-)|V|s\bar{s}\rangle} \to 0 \tag{37}$$

in the chiral limit, for which the ratio in eqn. 36 is $\sim -4/3$.

The ratio eqn. 36 depends sensitively on the pseudoscalar mixing angle and on a small breaking of chiral symmetry but remains negative in the range $-0.9 < \lambda < 0.5$. Thus we anticipate

$$r_0 \equiv \frac{\langle \eta\eta'|V|G_0\rangle}{\langle \eta\eta|V|G_0\rangle} = \frac{\langle gg|V|\eta'\rangle}{\langle gg|V|\eta\rangle} \sim -\frac{4}{3}. \tag{38}$$

There is some ambiguity as to how this is to be applied quantitatively since $m_{\eta'} \neq m_\eta$ and the wavefunctions at the origin $\psi_{\eta'}(0)$ and $\psi_\eta(0)$ are in general different. An alternative measure [52] may be the ratio $\Gamma(\psi \to \gamma\eta')/\Gamma(\psi \to \gamma\eta) = 5.0 \pm 0.6$. Dividing out phase space factors $\sim p_\gamma^3$ we obtain the ratio of matrix elements

$$r_0(J/\psi) = \pm(2.48 \pm 0.15). \tag{39}$$

The solution with the negative sign is compatible with a small breaking of chiral symmetry ($\lambda = 0.18$). This gives similar results to arguments based on the gluon anomaly in the pseudoscalar channel (see eqn. (60) in ref.[48] and also ref.[49]).

This enhanced gluonic production of $\eta\eta, \eta\eta'$ does not appear to be dramatic in the $\chi_{0,2}$ decays as the $\eta\eta/\pi\pi$ ratio appears to be "canonical" in the sense of section 5. This may be because $\langle gg|V|\eta \rangle$ and $\langle gg|V|\eta' \rangle$ are hidden in the large errors on the present data (in which case isolation of $\chi \to \eta'\eta'$ at a Tau Charm Factory would be especially interesting). Alternatively it may be $\langle gg|V|(\pi\pi)_s \rangle$ that is important and hence $\chi \to 4\pi$ is the signal. Indeed this channel is the biggest hadronic branching ratio for both χ_0 and χ_2. It would be interesting to compare the $\pi\pi$ spectra in these final states with those in $\eta_c \to \eta'\pi\pi$ and $\eta\pi\pi$ which are dominant modes in the η_c decays and may also be signals for this dynamics. High statistics from a Tau-Charm Factory may eventually answer this question. The relative coupling strengths of $\eta\pi\pi$ and $\eta'\pi\pi$ in decays of the glueball candidate $\eta(1420)$ are also relevant here.

Note that for the decays of $J^{PC} = 0^{++}, 2^{++} \ldots$ states, one will have a sharp test for a glueball if non-perturbative effects favour the direct $\eta\eta, \eta\eta'$ decay path over mixing with $Q\bar{Q}$ systems. A state decaying to $\eta\eta, \eta\eta'$ and/or $\eta'\eta'$ but not $\pi\pi$ nor $K\bar{K}$ cannot be simply $Q\bar{Q}$ since ρ and α cannot sensibly suppress $\pi\pi$ and $K\bar{K}$ simultaneously - (see e.g. fig. 1 and 3). GAMS has claimed states at 1590 MeV ($\eta\eta, \eta\eta'$) [15, 19], 1740 MeV ($\eta\eta$) [50] and 1910 MeV ($\eta\eta'$) [51] with no strong signal seen in $\pi\pi$ nor $K\bar{K}$. If the existence of any of these enigmatic states X is seen in other experiments such as central production $pp \to p(X)p$ or $\bar{p}p$ annihilation, where $X \to \eta\eta, \eta\eta'$ with no $K\bar{K}$ signal, this will be strong evidence for the presence of G_0 in their wavefunctions. The possible hints of $f_J(2100)$ in η channels [14] and of $f_2(2170)$ in $\eta\eta$ [42], if confirmed, will put a high premium on searching for or limiting the $K\bar{K}$ branching ratios for these states. A particular realisation of these generalities is the model of ref. [52]. Similar remarks apply to the decay of pseudoscalars $\to \eta(\pi\pi)_s$ in contrast to $\bar{K}(K\pi)_s$. This may be the case for the $\eta(1440)$ and for the η_c, as discussed above.

7 Application to Scalar Mesons around 1.5 GeV

7.1 Decays of $f_0(1500)$

We shall now combine these ideas with the other result of section 6, namely that at $O(V_1)$ the G_0 mixes with $Q\bar{Q}$ with amplitude ξ and that the resulting $Q\bar{Q}$ components decay as in section 3.

For simplicity of analysis we shall set $\rho = 1$. From eqn. 31 and 35 we obtain for $G \equiv f_0(1500)$:

$$\langle K\bar{K}|V_1|G \rangle = \frac{1 + \omega R^2}{2} \langle \pi\pi|V_1|G \rangle. \tag{40}$$

Eventual quantification of R_3 may be translated into a value of ωR^2 (see fig. 9). We shall scale all decay amplitudes relative to that for $\langle \pi\pi|V|G \rangle$ and see what this implies for the G_0 decay amplitudes. Thus

$$r_1 \equiv \frac{\langle \eta\eta|V|G\rangle}{\langle \pi\pi|V|G\rangle} = \frac{\langle \eta\eta|V|G_0\rangle}{N_G\langle \pi\pi|V|G\rangle} + (\frac{1+\omega R^2}{2}) + \cos2\phi(\frac{1-\omega R^2}{2}) = \pm(0.90 \pm 0.20) \quad (41)$$

from R_1 (eqn. 18) and

$$r_2 \equiv \frac{\langle \eta\eta'|V|G\rangle}{\langle \pi\pi|V|G\rangle} = \frac{\langle \eta\eta'|V|G_0\rangle}{N_G\langle \pi\pi|V|G\rangle} + \sin2\phi(\frac{1-\omega R^2}{2}) = \pm(0.53 \pm 0.11) \quad (42)$$

from R_2 (eqn. 19), from which we predict the ratio

$$r_0 = \frac{\langle \eta\eta'|V|G_0\rangle}{\langle \eta\eta|V|G_0\rangle} = \frac{2r_2 - \sin2\phi(1-\omega R^2)}{2r_1 - \cos\phi(1-\omega R^2) - 1 - \omega R^2}. \quad (43)$$

The ratio r_0 is plotted in fig. 10 as a function of ωR^2. Note that this ratio is rather sensitive to the precise value of the mixing angle ϕ. The solution with the $+$ sign in eqn. 41 and the $-$ sign in eqn. 42 (which we refer to as the "$+$ -" solution) agrees very well with ωR^2 in the range predicted by the current $K\overline{K}$ suppression (fig. 9) and with radiative J/ψ decay (eqn. 39). Figure 10 also suggests another possible solution ("- -") compatible with the $\psi \to \gamma\eta/\eta'$ ratio.

In the particular limit $\omega R^2 = -1$, the $\eta\eta$ decay mode would be driven dominantly by G_0 decay. The smaller rate for $\eta\eta'$ decay observed by Crystal Barrel would then be due to <u>destructive</u> interference between G_0 decay and the admixture of quarkonium in the wave function common to the "$+$ -" and "- -" solutions. The amplitudes for G_0 decay (fig. 6b) are given in appendix A and the invariant couplings shown in fig. 7b as a function of λ.

7.2 $f_0(1370)$

The decay amplitudes for $\Psi_n \to \pi\pi$ and $K\overline{K}$ will be those of an $n\bar{n}$ state such that,

$$\frac{\gamma^2(\Psi_n \to K\overline{K})}{\gamma^2(\Psi_n \to \pi\pi)} \simeq \frac{1}{3} \quad (44)$$

Assuming that the lower state Ψ_n is $f_0(1370)$ the decay to $\eta\eta'$ is kinematically forbidden and so the $\eta\eta$ decay will be the only one immediately sensitive to the predicted G_0 component in the $f_0(1370)$ Fock state. We find from the decay branching ratios measured by Crystal Barrel [9, 22] [3]

$$B(\bar{p}p \to f_0(1370)\pi^0, f_0 \to \pi^0\pi^0) < (2.6 \pm 0.4) \times 10^{-3},$$
$$B(\bar{p}p \to f_0(1370)\pi^0, f_0 \to \eta\eta) = (3.5 \pm 0.7) \times 10^{-4},$$

$$(45)$$

---

[3]The $<$ sign reflects the fact that the measured branching ratio for $f_0(1370)$ decay to $\pi\pi$ also includes some contribution from $f_0(980)$.

after phase space and form factor corrections:

$$\frac{\gamma^2(\Psi_n \to \eta\eta)}{\gamma^2(\Psi_n \to \pi\pi)} > 0.07 \tag{46}$$

or

$$\frac{\langle \Psi_n|V|\eta\eta\rangle}{\langle \Psi_n|V|\pi\pi\rangle} > 0.46. \tag{47}$$

On the other hand, eqn. 31 predicts

$$\frac{\langle \Psi_n|V|\eta\eta\rangle}{\langle \Psi_n|V|\pi\pi\rangle} = \cos^2\phi - \sqrt{2}\xi\frac{\langle G_0|V|\eta\eta\rangle}{\langle n\bar{n}|V|\pi\pi\rangle}, \tag{48}$$

which reduces to $\cos^2\phi = 0.63$ for a pure $n\bar{n}$ state. Solving for $\langle G_0|V|\eta\eta\rangle$ and introducing into eqn. 41 one finds with eqn. 47

$$\xi^2 = \frac{\cos^2\phi - 0.46}{2r_1 - (1 + \omega R^2) - \cos^2\phi(1 - \omega R^2)} \tag{49}$$

The result eqn.47 then implies that $|\xi|$ must be small and prefers the +- solution rather than the - - solution. One finds the 90% C.L. upper limit

$$|\xi| < 0.47 \tag{50}$$

which a posteriori justifies the first order perturbation used in the derivation of eqns. 27 and 31.

Further data analysis is now needed to quantify the experimental ratio eqn. 46 and compare it in detail with the above. In any event, the branching ratio of this $f_0(1370)$ state is consistent with $n\bar{n}$ dominance and hence further isolates the $f_0(1500)$ as an exceptional state.

7.3 The $f_0(1370)$ - $f_0(1500)$ system

It should by now be clear that it is the combination of the two siblings, $f_0(1370)$ and $f_0(1500)$, rather than either one on its own that reveals the need for degrees of freedom beyond $q\bar{q}$. We now illustrate how the branching ratios and widths of the pair manifest this quantitatively.

In the quark model of ref. [39] the widths of $^3P_0$ are qualitatively ordered as $\Gamma(\overline{n}n) > \Gamma(s\overline{s}) > \Gamma(a_0) \geq \Gamma(K^*)$. Empirically $\Gamma(a_0) = 270 \pm 40$ MeV, $\Gamma(K^*) = 287 \pm 23$ MeV which supports this pair to be members of the nonet and leads one to expect for their partners that $\Gamma(\overline{n}n) \sim 700$ MeV and $\Gamma(s\overline{s}) \sim 500$ MeV. In the flux tube model of ref.[44] (see also appendix B) after normalising to the known widths of the 2^{++} nonet one finds typically $\Gamma(K_0^*) > 200$ MeV and $\Gamma(a_0) > 300$MeV in accord with data, and predict $\Gamma(f_0^{n\bar{n}}) > 500$MeV. That the $f_0^{n\bar{n}}$ width will be very broad is a rather general conclusion of all standard quark models (see also ref.[53]); unitarisation effects do not alter this conclusion [32].

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The $f_0(1500)$ width of 116 ± 17 MeV is clearly out of line with this, being even smaller than the K^* and a_0 widths. The **total** width of $f_0(1370)$ is not yet well determined, 200-700 MeV being possible [9, 22] depending on the theoretical model used in the analysis.

These suppressions of widths are natural in the $G - Q\overline{Q}$ mixing scheme as the presence of the G_0 component dilutes the effect of the leading $n\bar{n}$ component:

$$\Gamma(\Psi_n) = \Gamma(n\bar{n})/(1 + 2\xi^2) > \frac{2}{3}\Gamma(n\bar{n}). \tag{51}$$

The $f(1500)$ by contrast is, in our hypothesis, a glueball in leading order and with $n\bar{n}, s\bar{s}$ components at $0(\xi)$ in perturbation. The decay amplitudes for $f(1500)$ are all at $0(\xi)$, as shown by our analysis above.

Quantitative measures arise if we concentrate on the decays into pseudoscalar pairs. The measurements from Crystal Barrel [9, 22] give the ratios of branching ratios for $G = f_0(1500)$ as

$$\frac{\eta\eta}{\pi\pi} = 0.23, \quad \frac{\eta\eta'}{\pi\pi} = 0.07 \tag{52}$$

which implies that $B(G \to \pi\pi) = 0.7F_2^G$ (where F_2^G is the fraction of of two-body decays). With $\Gamma_G = 116$ MeV this implies that $\Gamma(G \to \pi\pi) = 28.2F_2$ MeV per charge mode and hence, after dividing out phase space and form factors (738 MeV/c and 0.755 respectively) we have the reduced dimensionless measure

$$\tilde{\Gamma}(G \to \pi\pi) \equiv \frac{1}{3}\gamma^2(G \to \pi\pi) = 0.05F_2^G. \tag{53}$$

We now perform the same manipulations for the $f_0(1370)$. The $\pi\pi$ decay appears to be the dominant two-pseudoscalar decay mode (eqn. 45); any error on neglecting $\eta\eta$ in the analysis is likely to be masked anyway by the uncertainty on the total width which is not known to better than a factor of 3.5 (spanning 200 - 700 MeV). Dividing out the phase space (671 MeV/c) and form factors (0.798) as before, we form the reduced measure per charge combination

$$\tilde{\Gamma}(f(1370) \to \pi\pi) \equiv \frac{1}{3}\gamma^2(G \to \pi\pi) = (0.13 - 0.44)F_2^f. \tag{54}$$

Hence the ratio of measures for the two states is

$$\frac{\tilde{\Gamma}(G \to \pi\pi)}{\tilde{\Gamma}(f \to \pi\pi)} < 0.4\frac{F_2^G}{F_2^f}. \tag{55}$$

Assuming that $R = 1$ but allowing ω and ξ to be free, we expect this ratio to be given by

$$\frac{\tilde{\Gamma}(G \to \pi\pi)}{\tilde{\Gamma}(f \to \pi\pi)} = \frac{2\xi^2(1 + 2\xi^2)}{1 + \xi^2(2 + \omega^2)} \tag{56}$$

If F_2^f is not small, then indepandly of F_2^G:

$$|\xi| < 0.52, \tag{57}$$

which is consistent with our earlier, independent, estimate in eqn. 50. The sum of the partial widths of the two states for the $\pi\pi$ channels is

$$\Gamma(\Psi_n) + \Gamma(G) \simeq \Gamma(n\bar{n}). \tag{58}$$

Ref. [54] reports a strong ~ 300 MeV 4π signal in the 1400 MeV region. It is unlikely that this signal is $f_0(1370)$ since the inelasticity in the $\pi\pi$ S-wave would be very large ($\sim$ 80 %). However, given the uncertain dynamics in the $\pi\pi$ sector one must allow for this possibility in which case our result eqn. 57 would break down. On the other hand, a 4π contribution to $f_0(1500)$ decay would decrease the upper limit for $|\xi|$. A more detailed analysis is now warranted to verify if this further qualitative indication is supported and to quantify the resonant contributions F_2^G/F_2^f and their effect on the analysis above.

7.4 $f_0'(\text{``1600''})$

The mass of the Ψ_s state, the value of ωR^2 and of $m(G_0)$ are all related in our scheme and currently we can say no more than that the 1520 MeV to 1850 MeV range is possible for the mass of the $m(\Psi_s)$. A small value of $|\xi|$ would then require that Ψ_s decays essentially like an $s\bar{s}$ state, with couplings

$$\gamma^2(\pi\pi : K\bar{K} : \eta\eta : \eta\eta') = 0 : 4 : \frac{1}{2} : 2. \tag{59}$$

The decay of $f_0(1500)$ to $K\bar{K}$ will set the scale. The amount of G_0 mixing depends rather sensitively on the value of ω. However, a general result is that there will be <u>constructive</u> interference between G_0 and $s\bar{s}$ for the $\eta\eta'$ channel in either the "+-" or "--" solutions

$$\frac{\gamma^2(\Psi_s \to \eta\eta')}{\gamma^2(\Psi_s \to K\bar{K})} > \frac{\gamma^2(s\bar{s} \to \eta\eta')}{\gamma^2(s\bar{s} \to K\bar{K})} = \frac{1}{2}. \tag{60}$$

If we take the widths of $K^*(1430)$ and $a_0(1450)$ as a guide to normalize the width of the nonet in the Godfrey-Isgur model [39], we would anticipate $\Gamma(s\bar{s}) \sim 500$ MeV at a mass of 1600 MeV. The Ψ_s width will be suppressed relative to that of a pure $s\bar{s}$ due to the glueball component (reflected in the normalization, eqn. 31) but the actual branching ratios may be sensitively dependent on dynamics. In the model of ref. [44] the $K\bar{K}$ and $\eta\eta$ widths are suppressed if $m(\Psi_s) \to 1800 MeV$, see appendix B. The importance of the $\eta\eta'$ channel appears to be a solid prediction.

If the $f_J(1710)$ is confirmed to have a $J = 0$ componenet in $K\bar{K}$ but not in $\pi\pi$, this could be a viable candidate for a G_0- $s\bar{s}$ mixture, completing the scalar meson system built on the glueball and the quarkonium nonet.

Table 1: The scalar mesons, their observed decay modes and their suggested assignment.

Isospin	State	Decays	Nature
1	$a_0(980)$	$\eta\pi, K\overline{K}$	$K\overline{K}$ molecule
0	$f_0(980)$	$\pi\pi, K\overline{K}$	$K\overline{K}$ molecule
1	$a_0(1450)$	$\eta\pi$	$Q\overline{Q}$ 0^{++} nonet
0	$f_0(1370)$	$\pi\pi, \eta\eta, \rho\rho$	$Q\overline{Q}$ 0^{++} nonet
	$[f_0'(\text{``1600''})]$	$[K\overline{K}, \eta\eta, \eta\eta']$	$Q\overline{Q}$ 0^{++} nonet
1/2	$K_0^*(1430)$	$K\pi$	$Q\overline{Q}$ 0^{++} nonet
0	$f_0(1500)$	$\pi\pi, \eta\eta, \eta\eta', 4\pi^0$	glueball

8 The Scalar $Q\overline{Q}$ Mesons

Based on the analysis of the previous sections we shall assume that $f_0(1500)$ is mainly glue and shall examine whether a reasonable scalar $Q\overline{Q}$ nonet can be constructed with the other scalar mesons, neglecting first the small glue admixture in the two mainly $Q\overline{Q}$ isoscalars.

As we have seen, there are too many isoscalar 0^{++} mesons to fit in the $Q\overline{Q}$ ground state nonet. A possible classification of the scalar mesons is shown in table 1. The $a_0(980)$ and $f_0(980)$ are interpreted as $K\overline{K}$ molecules, a hypothesis that may be tested at DAΦNE [24]. The $a_0(1450)$, $f_0(1370)$, $f_0'(1600)$ and $K_0^*(1430)$ are the members of the ground state $Q\overline{Q}$ scalar nonet. Note that the masses of the strange and $I = 1$ members are similar; this is also the case for some other nonets, in particular 4^{++} [7].

The Crystal Barrel and Obelix Collaborations at LEAR also report the observation of a 0^{++} state decaying to $\rho^+\rho^-$ and $\rho^0\rho^0$ in $\overline{p}p$ annihilation at rest into $\pi^+\pi^-3\pi^0$ [54] and $\overline{p}n$ annihilation at rest in deuterium into $2\pi^+3\pi^-$ [55] (see also [56]). Given that mass (~ 1350 MeV) and width (~ 380 MeV) are compatible with $f_0(1370)$, one might assume that $f_0(1370)$ has been observed here in its $\rho\rho$ decay mode. The large $\rho\rho$ branching ratio points, however, to a large inelasticity in the $\pi\pi$ S-wave around 1400 MeV. Thus $f_0(1370)$ may split into two states, a $Q\overline{Q}$ decaying to $\pi\pi, K\overline{K}$ and $\eta\eta$ and another state decaying to $\rho\rho$, possibly a molecule [57].

The GAMS meson $f_0(1590)$ [19] decays to $\eta\eta'$ and $\eta\eta$ with a relative branching ratio of 2.7 ± 0.8 [7] which, given the large error, is consistent with eqn. 18 and 19 for $f_0(1500)$ (see footnote 1). We shall assume that $f_0(1590)$ is either identical to $f_0(1500)$ or that it is the predicted $s\overline{s}$ state. We are therefore left with one nonet, two to three molecules and the supernumerary $f_0(1500)$.

If the $s\overline{s}$ member lies at 1600 MeV then from the linear mass formula

$$\text{tg}^2\theta = \frac{4K_0^* - a_0 - 3f_0'}{3f_0 + a_0 - 4K_0^*}, \tag{61}$$

where the symbols denote the particle masses, one obtains for the $Q\overline{Q}$ nonet the singlet-octet mixing angle $(59^{+12}_{-7})°$ (or $121°$)). This is not far from ideal mixing, $\theta = 35.3°$ (or

$125.3°$) . The relative decay rates of the 0^{++} $Q\overline{Q}$ mesons to two pseudoscalars are given in appendix A and hence a further consistency check on this proposed nonet may be applied when the partial decay widths eventually become known. Note that the experimental widths for the known $Q\overline{Q}$ members of the proposed nonet (table 1) are in good agreement with predictions, see appendix B.

In the quark model, deviation from ideal mixing is due to mixing transitions between $u\overline{u}(d\overline{d})$ and $s\overline{s}$ which leads to a non-diagonal mass matrix in the flavour basis. The mixing energy (non-diagonal elements) is

$$3A = f_0 + f'_0 - 2K_0^* \tag{62}$$

which vanishes for ideal mixing. For our 0^{++} nonet one finds 37 MeV. The quark model also predicts the Schwinger sum rule

$$(f_0 + f'_0) \times (4K_0^* - a_0) - 3f_0 f'_0 - \frac{1}{3}(K^* - a_0)^2 = \frac{8}{3}(a_0 - K^*)^2 \times \cos^2\delta \tag{63}$$

where $\cos^2\delta$ is the fractional $Q\overline{Q}$ in the $f_0(1370)$ or f'_0("1600") wave function [58]. The fraction of glue in $f_0(1370)$ is according to eqn. 31:

$$\sin^2\delta = \frac{2\xi^2}{1 + 2\xi^2} < 0.33. \tag{64}$$

Figure 11 shows how the mixing angle δ in eqn. 64 varies as the function of the a_0 mass for various $f_0(1370)$ masses. A consistent result is indeed obtained when $m(f_0(1370)) \sim 1400$ MeV and $m(f_0(1600)) = 1600$ MeV. The fraction of glue in $f_0(1600)$ depends on ωR^2 (eqn. 31).

9 Conclusions

We have argued that the properties of the $f_0(1500) - f_0(1370)$ system are incompatible with them belonging to a quarkonium nonet. We suggest that $f_0(1500)$ is prominently a glueball mixed with the $\overline{n}n$ and $s\overline{s}$ quarkonia, the $f_0(1370)$ being dominantly $\overline{n}n$.

We have shown that a reasonable scalar nonet can be built with the newly discovered $a_0(1450)$ setting the mass scale, and its width, together with that of the $K^*(1430)$, setting the scale of the nonet widths which are within 25 % of those expected by earlier quark model calculations. The width of $f_0(1500)$ is dramatically suppressed relative to these and the width of the $f_0(1370)$ may be also somewhat suppressed. These results are in line with these two states being partners in a glueball - $Q\overline{Q}$ mixing scheme. It is this fortunate property that has enabled the $f_0(1500)$ to show up so prominently in several experiments where glueball channels are favoured.

Further supporting these arguments we have shown that there appears to be no dramatic intrinsic violation of flavour symmetry in decays involving gluons in a kinematic region where 0^{++} or 2^{++} glueball bound states are expected to be negligible. The flavour

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dependence of $f_0(1500)$ decays suggested that a significant mixing between G and $Q\bar{Q}$ states is distorting the branching ratios in the 0^{++} sector. We argued that the observed decay branching ratios could be due to the scalar glueball expected in this mass range, mixing with the two nearby $Q\bar{Q}$ isoscalars, one lying below, the other above $f_0(1500)$. The partial decay widths of the lower state, $f_0(1370)$, are consistent with a mainly $(u\bar{u} + d\bar{d})$ state. Our hypothesis also implies that the (mainly) $s\bar{s}$ state lies in the 1600 - 1700 MeV region.

The quantitative predictions of our analysis depend on the suppression of $f_0(1500)$ decay to $K\overline{K}$. Thus detailed study of $p\bar{p} \to \pi K\bar{K}$ can be seminal (i) in confirming the $K\overline{K}$ suppression, (ii) in confirming the $K(1430) \to K\pi$ and $a_0(1450) \to \eta\pi$ and $K\overline{K}$, (iii) in quantifying the signal for $f_0(1370)$ and $f_0(1500)$ and (iv) in isolating the predicted $s\bar{s}$ member of the nonet.

Clarifying the relationship between the Crystal Barrel $f_0(1500)$ and the $f_0(1590)$ of GAMS is important. Particular emphasis should be placed on the strength of the $\pi\pi$ branching ratio and the ratio of branching ratios $\eta\eta/\pi\pi$ in light of its potentially direct significance as a test for glueballs. If the $f_0(1550 \pm 50)$ becomes accepted as a scalar glueball, consistent with the predictions of the lattice, then searches for the 0^{-+} and especially the 2^{++} at mass 2.22 ± 0.13 GeV [5] may become seminal for establishing the lattice as a successful calculational laboratory.

Appendix A:
Amplitudes for arbitrary pseudoscalar mixing angles

The generalisations of the amplitudes for an arbitrary pseudoscalar mixing angle ϕ are as follows:

Quarkonium decay

$$
\begin{aligned}
\langle Q\overline{Q}|V|\pi\pi\rangle &= \cos\alpha \\
\langle Q\overline{Q}|V|K\overline{K}\rangle &= \cos\alpha(\rho - \sqrt{2}\tan\alpha)/2 \\
\langle Q\overline{Q}|V|\eta\eta\rangle &= \cos\alpha(\cos^2\phi - \rho\sqrt{2}\tan\alpha\sin^2\phi) \\
\langle Q\overline{Q}|V|\eta\eta'\rangle &= \cos\alpha\cos\phi\sin\phi(1 + \rho\sqrt{2}\tan\alpha)
\end{aligned}
\tag{65}
$$

for $I = 0$

$$
\begin{aligned}
\langle Q\overline{Q}|V|K\overline{K}\rangle &= \rho/2 \\
\langle Q\overline{Q}|V|\pi\eta\rangle &= \cos\phi \\
\langle Q\overline{Q}|V|\eta\eta'\rangle &= \sin\phi
\end{aligned}
\tag{66}
$$

for $I = 1$ and

$$
\begin{aligned}
\langle Q\overline{Q}|V|K\pi\rangle &= \sqrt{3}/2 \\
\langle Q\overline{Q}|V|K\eta\rangle &= (\rho\sin\phi - \cos\phi/\sqrt{2})/\sqrt{2} \\
\langle Q\overline{Q}|V|K\eta'\rangle &= (\rho\cos\phi + \sin\phi/\sqrt{2})/\sqrt{2}
\end{aligned}
\tag{67}
$$

for $I = 1/2$.

$G \rightarrow QQ\bar{Q}\bar{Q}$ (graph 6a)

$$
\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \\
\langle G|V|K\overline{K}\rangle &= R \\
\langle G|V|\eta\eta\rangle &= \cos^2\phi + R^2\sin^2\phi \\
\langle G|V|\eta\eta'\rangle &= \cos\phi\sin\phi(1 - R^2)
\end{aligned}
\tag{68}
$$

$G \to GG$ (graph 6b)

$$\begin{aligned}
\langle G|V|\pi\pi\rangle &= 0 \\
\langle G|V|K\overline{K}\rangle &= 0 \\
\langle G|V|\eta\eta\rangle &= (\cos\phi\sqrt{2} - \lambda\sin\phi)^2 \\
\langle G|V|\eta\eta'\rangle &= (\cos\phi\sqrt{2} - \lambda\sin\phi)(\sin\phi\sqrt{2} + \lambda\cos\phi)
\end{aligned}$$

(69)

where

$$\lambda \equiv \frac{\langle G(0^-)|V|d\bar{d}\rangle}{\langle G(0^-)|V|s\bar{s}\rangle} \tag{70}$$

in the pseudoscalar channel.

$G \to Q\overline{Q}$ (graph 6c)

$$\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \\
\langle G|V|K\overline{K}\rangle &= (\rho + \omega R^2)/2 \\
\langle G|V|\eta\eta\rangle &= (\cos^2\phi + \omega\rho R^2\sin^2\phi) \\
\langle G|V|\eta\eta'\rangle &= \cos\phi\sin\phi(1 - \omega\rho R^2)
\end{aligned}$$

(71)

Appendix B:
Form Factors

In the main body of the text we used rather simple forms for form factors, eqs (14). In order to test sensitivity to these assumptions we consider here the consequences of more structured form factors as arise when the dynamical effects of flux-tube breaking are included. The result is that momentum dependent multiplicative factors enter additional to those already in eqn (14); the general structure is discussed in table 2 and appendix B of ref[44]. In the approximation where the light hadron wavefunctions have the same scale parameter (the quantity β in eqn (14)) the structure of $S-$wave decay amplitudes (as for $^3P_0 \rightarrow{}^1 S_0 +{}^1 S_0$) is

$$S = (1 - \frac{2q^2}{9\beta^2})\exp(-\frac{q^2}{12\beta^2})$$

The factor $\frac{q^2}{\beta^2}$ in parenthesis, which was not present before, arises from the coupling of the $^3P_0$ of the initial $Q\bar{Q}$ meson with the $^3P_0$ of the $q\bar{q}$ pair created by the breaking of the flux-tube and which seed the decay. From detailed fits to meson spectroscopy and decays it is known that $\beta^{-2} \simeq 5$ to 6 GeV^{-2} and so the multiplicative factor $(1 - \frac{2q^2}{9\beta^2})$ does not significantly affect our analysis of the $f_0(1500)$.

In this appendix we have followed ref[44] in imposing $\beta(^3P_0) = 0.5$ GeV and have used a more modern value [59] of $\beta = 0.4$ GeV for the $^1S_0$ and $^3P_2$ states. Barnes et al [53] have made a similar analysis with $\beta \simeq 0.4$ GeV as a preferred overall value and find similar results. The strategy is to use the known $^3P_2$ decays to set the overall scale following the prescriptions in table 2 and appendix B of ref.[44].

(1) $f_0^n(1370)$

Using $\Gamma(f_2 \rightarrow \pi\pi) = 157$ MeV yields, in MeV

$$
\begin{aligned}
\Gamma(f_0^n \rightarrow \pi\pi) \quad &= 270 \pm 25 \\
K\bar{K} \quad &= 195 \pm 20 \\
\eta\eta \quad &= 95 \pm 10
\end{aligned}
$$

where the η is assumed to be a 50:50 mixture of $s\bar{s}$ and $n\bar{n}$. The $(1 - \frac{2q^2}{9\beta^2})$ factor has suppressed the $\pi\pi$ more markedly than the $K\bar{K}$ and $\eta\eta$, hence leading to a larger $\eta\eta/\pi\pi$ and $K\bar{K}/\pi\pi$ ratio than in the main text. Nonetheless one sees that the $\Gamma(f_0^n) >> \Gamma(f_2^n)$ still arises in line with the conclusion that a "narrow" f_0 width is out of line with a $^3P_0$ quarkonium state. This conclusion is reinforced by the expectation based on spin counting arguments that $\gamma^2(\rho\rho) > \gamma^2(\pi\pi)$ and hence that there should be a non-negligible $\Gamma(f_0^n \rightarrow \rho(\pi\pi)_p)$ in addition to the two-body channels.

(2) $K_0^*(1430)$

Using $\Gamma(K_2^* \to K\pi) \simeq 50$ MeV or $\Gamma(f_2)$ as above yields consistent similar results, namely in MeV

$$
\begin{aligned}
\Gamma(K_0^* \to K\pi) &= 200 \pm 20 \\
K\eta &\simeq 0 \\
K\eta' &= 15 \pm 20
\end{aligned}
$$

The $K\eta'$ has a large coupling for physical $\eta\eta'$ mixing angles but the width is very sensitive to phase space. We note that the Particle Data Group[7] allow $(7\pm10)\%$ "non-$K\pi$" for the K_0^* decay and we have assigned this to $K\eta'$.

These results should be compared with the experimental value $\Gamma(K_0^*) = 287\pm23$ MeV.

(3) $a_0(1450)$

Using $\Gamma(a_2 \to K\bar{K}) = 5.2\pm0.9$ MeV [7] we obtain for the corresponding channel $\Gamma(a_0 \to K\bar{K}) \simeq 110$ MeV. Then with physical $\eta\eta'$ mixing angles we have $\Gamma(\eta\pi) \simeq 90$ MeV, $\Gamma(\eta'\pi) \simeq 80$ MeV. There is considerable uncertainty in these widths, however, since if we were to normalise by $\Gamma(f_2 \to \pi\pi)$ instead of by $\Gamma(a_2 \to K\bar{K})$ we would find $\Gamma(a_0 \to KK : \eta\pi : \eta'\pi) \simeq 200:160:140$ MeV. Reflecting these uncertainties we can merely summarise by

$$
\Gamma(a_0 \to 0^-0^-)_{theory} = 390 \pm 110 \ MeV
$$
$$
\Gamma(a_0)_{\exp} = 270 \pm 40 \ MeV.
$$

The general conclusion that $\Gamma(f_0^n) > \Gamma(a_0) \geq \Gamma(K_0^*)$ holds true. Furthermore we note that these results require that if the $a_0(1450)$ seen by Crystal Barrel is indeed $^3P_0$ $(Q\bar{Q})$, then comparable partial widths are expected for all of $K\bar{K}, \eta\pi$ and $\eta'\pi$. Quantifying these experimentally will be an important piece of the total strategy in clarifying the nature of these scalar mesons.

(4) $f_0^s(1.6\text{-}1.8)$

The $(1 - \frac{2q^2}{9\beta^2})$ can have a dramatic effect in the upper part of this range of masses. Using the $\Gamma(f_2^n \to \pi\pi)$ or $\Gamma(f_2^s \to K\bar{K})$ as normalisation, the mass dependence of the partial widths of a pure $^3P_0(s\bar{s})$ are as follows, in MeV

	1600	1700	1800
$\Gamma(f_0^s \to KK)$	270	155	85
$\eta\eta$		45	25	20
$\eta\eta'$		195	170	190

The effect of the momentum node is clearly seen in the KK and $\eta\eta$ channels, whereas the $\eta\eta'$ maintains its strength. There is nothing in this pattern of widths that supports a $^3P_0(Q\bar{Q})$ interpretation of $f_0(1500)$. A detailed study of quarkonium decays with similar conclusions is in ref[53].

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Figure Captions

Figure 1: γ_{ij}^2 as a function of α for $\rho = 1$ (a) and $\rho = 0.75$ or 1.25 (b) for quarkonium decay (up to a common multiplicative factor). Dotted line: $\pi\pi$; dash-dotted line: $K\overline{K}$; dashed line: $\eta\eta'$; solid line: $\eta\eta$.

Figure 2: χ^2 for 2^{++} quarkonium decay as a function of ρ for various values of β. The 5% C.L. limit is shown by the horizontal line.

Figure 3: $\eta\eta/\pi\pi$ vs $\eta\eta/K\overline{K}$ invariant couplings per charge combination for various values of $\rho \leq 1$. The grey region where both ratios are larger than one is not accessible to $Q\overline{Q}$ mesons unless $s\overline{s}$ production is enhanced.

Figure 4: $\rho\tan(\alpha)$ as a function of $\eta\eta/\pi\pi$ (full curve) and $\eta\eta'/\pi\pi$ (dotted curve). The vertical arrows show the experimental ranges of R_1 and R_2 for $f_0(1500)$ decay, left with form factor, right without form factor.

Figure 5: ρ as a function of α. The full curves show the 5 and 10 % C.L. upper (+) and lower(-) limits dependence for the experimental value R_1, the dashed curves for R_2. The dotted curves give the boundaries for the experimental value R_3. The grey region shows the range allowed by the experimental data on $f_0(1500)$ decay to $\pi\pi$, $\eta\eta$ and $\eta\eta'$. The black region includes in addition the bubble chamber upper limit for $K\overline{K}$.

Figure 6: Contributions to gluonium decay: $QQ\bar{Q}Q$ (a), GG (b), $Q\overline{Q}$ (c) and interpretation as $Q\bar{Q}$ mixing (d) involving the energy denominator $E_G - E_{Q\bar{Q}}$

Figure 7: Predicted decay rates γ_{ij}^2 (up to a common multiplicative constant) (a) as a function of R^2 for $QQ\overline{Q}Q$ decay, (b) as a function of λ for GG decay and (c) as a function of ωR^2 for $Q\overline{Q}$ decays with $\rho = 1$. Solid line: $\pi\pi$; dashed line: $K\overline{K}$; dash-dotted line: $\eta\eta$; dotted line: $\eta\eta'$.

Figure 8: Glueballs, quarkonia and perturbations: (a) primitive $Q\bar{Q}$ and (b) primitive glueball G_0 in flux tube simulation of lattice QCD; perturbation V_1 (c) and V_2 (d); the effect of V_1 on $Q\bar{Q}$ is shown in (e), and on G is shown in (f); the effect of V_2 on G is shown in (g).

Figure 9: R_3 versus ωR^2.

Figure 10: Predicted ratio r_0 as a function of ωR^2 for $f_0(1500)$ decays. The solid lines show the four possible solutions from the Crystal Barrel results r_1 and r_2 with the corresponding signs. The range allowed by the experimental errors is shown by the dashed lines for the "+ -" and "- -" solution. The solutions "+ -" and "- -" are compatible with ωR^2 in the range allowed by the $K\overline{K}$ suppression and radiative J/ψ decay (area between the parallel lines, indicated by the grey flashes). A small breaking of chiral symmetry favors the "+ -" solution.

Figure 11: Fractional contribution of $Q\overline{Q}$ in the $f_0(1370)$ wave function from the Schwinger sum rule as a function of a_0 mass for various $f_0(1370)$ masses, assuming the (mainly) $s\bar{s}$ state to lie at 1600 MeV. The horizontal arrow shows the experimental uncertainty in the a_0 mass and the vertical arrow shows the range allowed for $|\xi| < 0.5$.